Chapter 5
Determination of Forward and Futures Prices

Consumption vs Investment Assets
- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)

Short Selling (Page 102-103)
- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way
Short Selling (continued)

- At some stage you must buy the securities so they can be replaced in the account of the client.
- You must pay dividends and other benefits the owner of the securities receives.
- There may be a small fee for borrowing the securities.

Example

- You short 100 shares when the price is $100 and close out the short position three months later when the price is $90.
- During the three months a dividend of $3 per share is paid.
- What is your profit?
- What would be your loss if you had bought 100 shares?

Notation for Valuing Futures and Forward Contracts

- $S_0$: Spot price today
- $F_0$: Futures or forward price today
- $T$: Time until delivery date
- $r$: Risk-free interest rate for maturity $T$
An Arbitrage Opportunity?

Suppose that:
- The spot price of a non-dividend-paying stock is $40
- The 3-month forward price is $43
- The 3-month US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

Another Arbitrage Opportunity?

Suppose that:
- The spot price of non-dividend-paying stock is $40
- The 3-month forward price is US$39
- The 1-year US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

The Forward Price

If the spot price of an investment asset is $S_0$ and the futures price for a contract deliverable in $T$ years is $F_0$, then

$$F_0 = S_0 e^{rT}$$

where $r$ is the $T$-year risk-free rate of interest.

In our examples, $S_0 = 40$, $T = 0.25$, and $r = 0.05$ so that

$$F_0 = 40 e^{0.05 \times 0.25} = 40.50$$
If Short Sales Are Not Possible.

Formula still works for an investment asset because investors who hold the asset will sell it and buy forward contracts when the forward price is too low.

When an Investment Asset Provides a Known Income (page 107, equation 5.2)

\[ F_0 = (S_0 - I)e^{rT} \]

where \( I \) is the present value of the income during life of forward contract.

When an Investment Asset Provides a Known Yield (Page 109, equation 5.3)

\[ F_0 = S_0 e^{(r-q)T} \]

where \( q \) is the average yield during the life of the contract (expressed with continuous compounding).
End-of-Chapter Questions

Problem 5.4: A stock index currently stands at 350. The risk-free interest rate is 8% per annum (with continuous compounding) and the dividend yield on the index is 4% per annum. What should the futures price for a four-month contract be?

The futures price is $350e^{0.08 \cdot 0.3333 - 0.04 \cdot 0.3333} = $354.7

Valuing a Forward Contract

A forward contract is worth zero (except for bid-offer spread effects) when it is first negotiated.
Later it may have a positive or negative value.
Suppose that $K$ is the delivery price for a contract that would be negotiated today and $F_0$ is the forward price.
As time passes, $K$ stays the same (because it's part of the definition of the contract), but the forward price changes and the value of the contract becomes either positive or negative.

Valuing a Forward Contract

By considering the difference between a contract with delivery price $K$ and a contract with delivery price $F_0$, we can deduce that:

- the value of a long forward contract, $f$, is $(F_0 - K)e^{-rT}$
- the value of a short forward contract is $(K - F_0)e^{-rT}$
Problem 5.9: A one-year long forward contract on a non-dividend-paying stock is entered into when the stock price is $40 and the risk-free rate of interest is 10% per annum with continuous compounding.

a) What are the forward price and the initial value of the forward contract?

b) Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

- The forward price, , is given by equation (5.1) as: \( F_e = 40e^{0.10} = 44.21 \) or $44.21. The initial value of the forward contract is zero.

- Six months later, the price of the stock is $45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract?

The delivery price in the contract is $44.21. The value of the contract, , after six months is given by equation (5.5) as: \( f_e = 45e^{0.10\times 0.5} = 47.31 \) i.e., it is $2.95. The forward price is: \( 45e^{0.10\times 0.5} = 47.31 \)

Forward vs Futures Prices

- When the maturity and asset price are the same, forward and futures prices are usually assumed to be equal. (Eurodollar futures are an exception)

- When interest rates are uncertain they are, in theory, slightly different:
  - A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
  - A strong negative correlation implies the reverse
**Stock Index** (Page 112-114)

- Can be viewed as an investment asset paying a dividend yield.
- The futures price and spot price relationship is therefore:
  \[ F_0 = S_0 e^{(r-q)T} \]
  where \( q \) is the average dividend yield on the portfolio represented by the index during life of contract.

**Example: Stock Index**

- Consider a 3-month futures contract on an index. Suppose that the stocks underlying the index provide a dividend yield of 1% per annum, that the current value of the index is 1300 and that the continuously compounded risk-free interest rate is 5% per annum. What is \( F_0 \)?
  \[ F_0 = S_0 e^{(r-q)T} = 1300 \cdot e^{(0.05-0.01)\cdot0.25} = 1313.07 \]

**Stock Index** (continued)

- For the formula to be true it is important that the index represent an investment asset.
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio.
- The Nikkei index viewed as a dollar number does not represent an investment asset (See Business Snapshot 5.3, page 113).
**Index Arbitrage**

- When $F_0 > S_0 e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures.
- When $F_0 < S_0 e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index.

**Index Arbitrage (continued)**

- Index arbitrage involves simultaneous trades in futures and many different stocks.
- Very often a computer is used to generate the trades.
- Occasionally simultaneous trades are not possible and the theoretical no-arbitrage relationship between $F_0$ and $S_0$ does not hold (see Business Snapshot 5.4 on page 114).

**Futures and Forwards on Currencies** (*Page 112-115*)

- A foreign currency is analogous to a security providing a yield.
- The yield is the foreign risk-free interest rate.
- It follows that if $r_f$ is the foreign risk-free interest rate:
  \[ F_0 = S_0 e^{(r_f - r_q)T} \]
Example: Currency

Suppose that 2-year interest rates in Australia and the US are 5% and 7%, respectively, and the spot exchange rate between the Australian dollar (AUD) and the US dollar (USD) is 0.62 USD per AUD. What should be the 2-year forward exchange rate?

\[ F_0 = S_0 e^{(r_f - r)T} = 0.62 e^{(0.07 - 0.05) \times 2} = 0.6453 \]

Consumption Assets: Storage is Negative Income

\[ F_0 \leq S_0 e^{(r + u)T} \]

where \( u \) is the storage cost per unit time as a percent of the asset value.

Alternatively,

\[ F_0 \leq (S_0 + U)e^{rT} \]

where \( U \) is the present value of the storage costs.

End-of-Chapter Questions

Problem 5.1: Explain what happens when an investor shorts a certain share.

The investor’s broker borrows the shares from another client’s account and sells them in the usual way. To close out the position, the investor must purchase the shares. The broker then replaces them in the account of the client from whom they were borrowed. The party with the short position must remit to the broker dividends and other income paid on the shares. The broker transfers these funds to the account of the client from whom the shares were borrowed.
End-of-Chapter Questions

Problem 5.2: What is the difference between the forward price and the value of a forward contract?

The forward price of an asset today is the price at which you would agree to buy or sell the asset at a future time.

The value of a forward contract is zero when you first enter into it. As time passes the underlying asset price changes and the value of the contract may become positive or negative.

End-of-Chapter Questions

Problem 5.3: Suppose that you enter into a six-month forward contract on a non-dividend-paying stock when the stock price is $30 and the risk-free interest rate (with continuous compounding) is 12% per annum. What is the forward price?

The forward price is $31.86

End-of-Chapter Questions

Problem 5.7: Explain why a foreign currency can be treated as an asset providing a known yield.

A foreign currency provides a known interest rate, but the interest is received in the foreign currency. The value in the domestic currency of the income provided by the foreign currency is therefore known as a percentage of the value of the foreign currency. This means that the income has the properties of a known yield.